

### § 1.3 数列の項の総和の公式

#### 問題 1.3.1

$$\begin{aligned}\sum_{k=1}^{30} \frac{5k-7}{6} &= \frac{1}{6} \left( 5 \sum_{k=1}^{30} k - \sum_{k=1}^{30} 7 \right) = \frac{1}{6} \left( 5 \cdot \frac{30 \cdot 31}{2} - 7 \cdot 30 \right) = \frac{30}{6} \left( \frac{5 \cdot 31}{2} - 7 \right) = 5 \left( \frac{155}{2} - 7 \right) \\ &= \frac{705}{2} .\end{aligned}$$

#### 問題 1.3.2

$$\begin{aligned}\sum_{j=1}^{20} \frac{j(3j-7)}{10} &= \frac{1}{10} \left( 3 \sum_{j=1}^{20} j^2 - 7 \sum_{j=1}^{20} j \right) = \frac{1}{10} \left( 3 \cdot \frac{20 \cdot 21 \cdot 41}{6} - 7 \cdot \frac{20 \cdot 21}{2} \right) \\ &= \frac{20 \cdot 21}{10} \cdot \left( 3 \cdot \frac{41}{6} - 7 \cdot \frac{1}{2} \right) = 42 \cdot \frac{102}{6} = 7 \cdot 102 \\ &= 714 .\end{aligned}$$

#### 問題 1.3.3

$$\begin{aligned}\sum_{i=0}^n \frac{4i^2-3}{5} &= \frac{1}{5} \left( 4 \sum_{i=0}^n i^2 - \sum_{i=0}^n 3 \right) = \frac{1}{5} \left\{ 4 \cdot \frac{n(n+1)(2n+1)}{6} - 3(n+1) \right\} \\ &= \frac{n+1}{5} \left\{ \frac{2n(2n+1)}{3} - 3 \right\} = \frac{n+1}{5} \frac{4n^2+2n-9}{3} \\ &= \frac{(n+1)(4n^2+2n-9)}{15} .\end{aligned}$$

#### 問題 1.3.4

$$\begin{aligned}\sum_{j=1}^n \frac{j(3j-4)}{5} &= \frac{1}{5} \sum_{j=1}^n (3j^2-4j) = \frac{1}{5} \left( 3 \sum_{j=1}^n j^2 - 4 \sum_{j=0}^n j \right) = \frac{1}{5} \left\{ 3 \cdot \frac{n(n+1)(2n+1)}{6} - 4 \cdot \frac{n(n+1)}{2} \right\} \\ &= \frac{n(n+1)}{5} \left( \frac{2n+1}{2} - 2 \right) = \frac{n(n+1)}{5} \frac{2n-3}{2} \\ &= \frac{n(n+1)(2n-3)}{10} .\end{aligned}$$