## §1.5　等比数列

問題 1.5.1　数列 $\{a_n\}_{n\geq 0}$ は公比 $\frac{1}{3}$ の等比数列なので,

$$a_n = a_2\left(\frac{1}{3}\right)^{n-2} = 45\frac{1}{3^{n-2}} = \frac{5\cdot 3^2}{3^{n-2}} = \frac{5}{3^{n-4}} \ .$$

問題 1.5.2　等比数列 $\{a_n\}_{n\geq 0}$ の公比を $r$ とおく.　$a_{12} = a_3 r^{12-3} = a_3 r^9$　なので, $r^9 = \frac{a_{12}}{a_3} = \frac{500}{4} = 125$ , $r > 0$ なので $r = 125^{\frac{1}{9}} = (5^3)^{\frac{1}{9}} = 5^{\frac{1}{3}}$ . 従って,

$$a_n = a_3 r^{n-3} = 4\cdot\left(5^{\frac{1}{3}}\right)^{n-3} = 4\cdot 5^{\frac{n}{3}-1} \ .$$

問題 1.5.3　$S = \sum_{k=3}^{n}(8\cdot 5^k)$ とおく.

$$S = 8\cdot 5^3 + 8\cdot 5^4 + 8\cdot 5^5 + 8\cdot 5^6 + \cdots + 8\cdot 5^{n-1} + 8\cdot 5^n$$
$$5S = 8\cdot 5^4 + 8\cdot 5^5 + 8\cdot 5^6 + 8\cdot 5^7 + \cdots + 8\cdot 5^n + 8\cdot 5^{n+1}$$

$5S$ から $S$ を引くと $4S = 8\cdot 5^{n+1} - 8\cdot 5^3$ , $S = 2\cdot 5^{n+1} - 2\cdot 5^3 = 2\cdot 5^{n+1} - 250$ . よって $\sum_{k=3}^{n}(8\cdot 5^k) = 2\cdot 5^{n+1} - 250$ .

問題 1.5.4　$S = \sum_{k=1}^{n}\left\{4\left(\frac{3}{5}\right)^k\right\}$ とおく.

$$S = 4\cdot\frac{3}{5} + 4\left(\frac{3}{5}\right)^2 + 4\left(\frac{3}{5}\right)^3 + 4\left(\frac{3}{5}\right)^4 + \cdots + 4\left(\frac{3}{5}\right)^{n-1} + 4\left(\frac{3}{5}\right)^n ,$$
$$\frac{5}{3}S = 4 + 4\cdot\frac{3}{5} + 4\left(\frac{3}{5}\right)^2 + 4\left(\frac{3}{5}\right)^3 + \cdots + 4\left(\frac{3}{5}\right)^{n-2} + 4\left(\frac{3}{5}\right)^{n-1} ,$$

$\frac{5}{3}S$ から $S$ を引くと $\frac{2}{3}S = 4 - 4\left(\frac{3}{5}\right)^n$ なので, $S = 6 - 6\left(\frac{3}{5}\right)^n$ , つまり $\sum_{k=1}^{n}\left\{4\left(\frac{3}{5}\right)^k\right\} = 6 - 6\left(\frac{3}{5}\right)^n$ .

問題 1.5.5

$$\sum_{k=1}^{n}\left\{6\left(\frac{5}{3}\right)^n\right\} = \frac{10\left\{\left(\frac{5}{3}\right)^n - 1\right\}}{\frac{5}{3} - 1} = 15\left\{\left(\frac{5}{3}\right)^n - 1\right\} \ .$$

問題 1.5.6

$$4 + 12 + 36 + 108 + 324 + \cdots + 4\cdot 3^n = \sum_{k=1}^{n+1}(4\cdot 3^{k-1}) = \frac{4(3^{n+1}-1)}{3-1} = 2(3^{n+1}-1) \ .$$