

§ 2.4 関数の極限の計算

問題 2.4.1

$$\lim_{u \rightarrow -2} \frac{u^2 - u - 6}{u + 2} = \lim_{u \rightarrow -2} \frac{(u+2)(u-3)}{u+2} = \lim_{u \rightarrow -2} (u-3) = -2-3 = -5.$$

問題 2.4.2

$$\lim_{x \rightarrow 3} \frac{2x^2 - 5x - 3}{x^2 - 5x + 6} = \lim_{x \rightarrow 3} \frac{(2x+1)(x-3)}{(x-2)(x-3)} = \lim_{x \rightarrow 3} \frac{2x+1}{x-2} = \frac{\lim_{x \rightarrow 3} (2x+1)}{\lim_{x \rightarrow 3} (x-2)} = \frac{7}{1} = 7.$$

問題 2.4.3

$$\lim_{t \rightarrow 1} \frac{t^2 - 3t + 2}{t^2 - 2t + 3} = \frac{\lim_{t \rightarrow 1} (t^2 - 3t + 2)}{\lim_{t \rightarrow 1} (t^2 - 2t + 3)} = \frac{0}{2} = 0.$$

問題 2.4.4

$x = \frac{t}{3}$ とおく. $t = 3x$. $t \rightarrow 0$ のとき $x \rightarrow 0$ なので、

$$\lim_{t \rightarrow 0} \frac{\sin \frac{t}{3}}{t} = \lim_{x \rightarrow 0} \frac{\sin x}{3x} = \frac{1}{3} \lim_{x \rightarrow 0} \frac{\sin x}{x} = \frac{1}{3} \cdot 1 = \frac{1}{3}.$$

問題 2.4.5

$$\lim_{y \rightarrow 0} \frac{\sin(y + \pi)}{y} = \lim_{y \rightarrow 0} \frac{-\sin y}{y} = -\lim_{y \rightarrow 0} \frac{\sin y}{y} = -1.$$

問題 2.4.6

$t = \frac{x}{5}$ とおく. $x = 5t$.

$$\frac{\tan \frac{x}{5}}{x} = \frac{\tan t}{5t} = \frac{1}{5} \frac{\sin t}{\cos t} = \frac{1}{5} \frac{\sin t}{t} \frac{1}{\cos t}.$$

$x \rightarrow 0$ のとき $t \rightarrow 0$. $\lim_{t \rightarrow 0} \frac{\sin t}{t} = 1$ なので、

$$\lim_{x \rightarrow 0} \frac{\tan \frac{x}{5}}{x} = \lim_{t \rightarrow 0} \left(\frac{1}{5} \frac{\sin t}{t} \frac{1}{\cos t} \right) = \frac{1}{5} \cdot \lim_{t \rightarrow 0} \frac{\sin t}{t} \cdot \frac{1}{\lim_{t \rightarrow 0} \cos t} = \frac{1}{5} \cdot 1 \cdot \frac{1}{1} = \frac{1}{5}.$$