

§ 2.8 三角関数の微分係数

問題 2.8

$$f(a+h) - f(a) = \cos\{3(a+h)\} - \cos 3a = -2 \sin \frac{3(2a+h)}{2} \sin \frac{3h}{2} = -2 \sin \left(3a + \frac{3h}{2}\right) \sin \frac{3h}{2} .$$

変数 x を $x = \frac{3h}{2}$ とおく． $h = \frac{2x}{3}$ なので，

$$\frac{f(a+h) - f(a)}{h} = \frac{-2 \sin \left(3a + \frac{3h}{2}\right) \sin \frac{3h}{2}}{h} = -\frac{2 \sin(3a+x) \sin x}{\frac{2x}{3}} = -3 \sin(3a+x) \frac{\sin x}{x} .$$

$h \rightarrow 0$ のとき $x = \frac{3h}{2} \rightarrow 0$. 正弦関数 $\sin x$ は連続なので $\lim_{x \rightarrow 0} \sin(3a+x) = \sin \left\{ \lim_{x \rightarrow 0} (3a+x) \right\} =$

$\sin(3a)$. $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$. a における f の微分係数は

$$\begin{aligned} \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} &= \lim_{x \rightarrow 0} \left\{ -3 \sin(3a+x) \frac{\sin x}{x} \right\} \\ &= -3 \left\{ \lim_{x \rightarrow 0} \sin(3a+x) \right\} \cdot \lim_{x \rightarrow 0} \frac{\sin x}{x} = -3 \sin(3a) \cdot 1 \\ &= -3 \sin 3a . \end{aligned}$$