

### § 3.3 関数の積・商の微分法

#### 問題 3.3.1

$$\begin{aligned} f'(x) &= \frac{d}{dx}(\sin x \cos x) = \frac{d}{dx} \sin x \cdot \cos x + \sin x \frac{d}{dx} \cos x = \cos x \cos x + \sin x (-\sin x) \\ &= \cos^2 x - \sin^2 x . \end{aligned}$$

#### 問題 3.3.2

$$\psi'(x) = \frac{d}{dx} \frac{2}{3 \sin x + 4} = -\frac{2 \frac{d}{dx}(3 \sin x + 5)}{(3 \sin x + 4)^2} = -\frac{2 \cdot 3 \frac{d}{dx} \sin x}{(3 \sin x + 4)^2} = -\frac{6 \cos x}{(3 \sin x + 4)^2} .$$

#### 問題 3.3.3

$$\begin{aligned} \frac{dv}{du} &= \frac{d}{du} \frac{\cos u}{u^4} = \frac{\frac{d}{du} \cos u \cdot u^4 - \cos u \cdot \frac{d}{du} u^4}{(u^4)^2} = \frac{-\sin u \cdot u^4 - \cos u \cdot 4u^3}{u^8} \\ &= -\frac{u \sin u + 4 \cos u}{u^5} . \end{aligned}$$

#### 問題 3.3.4

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \frac{5x-3}{x^2-4} = \frac{\frac{d}{dx}(5x-3) \cdot (x^2-4) - (5x-3) \frac{d}{dx}(x^2-4)}{(x^2-4)^2} = \frac{5(x^2-4) - (5x-3)2x}{(x^2-4)^2} \\ &= -\frac{5x^2-6x+20}{(x^2-4)^2} . \end{aligned}$$

#### 問題 3.3.5

$$f'(x) = \frac{d}{dx}(x^3 \ln x) = \frac{d}{dx} x^3 \cdot \ln x + x^3 \frac{d}{dx} \ln x = 3x^2 \ln x + x^3 \frac{1}{x} = x^2(1 + 3 \ln x) .$$

$e^4$  における  $f$  の微分係数は、

$$f'(e^4) = (e^4)^2(1 + 3 \ln e^4) = e^8(1 + 3 \cdot 4) = 13e^8 .$$