

§ 3.5 合成関数の微分法

問題 3.5.1 $t = 5x - 2$ とおく.

$$\begin{aligned}\frac{dy}{dx} &= \frac{d}{dx} \cos(5x - 3) = \frac{d}{dt} \cos t = \frac{d}{dt} \cos t \cdot \frac{dt}{dx} = -\sin t \cdot \frac{d}{dx}(5x - 3) = -\sin(5x - 3) \cdot 5 \\ &= -5 \sin(5x - 3) .\end{aligned}$$

問題 3.5.2 $t = \frac{7x - 5}{3}$ とおく. $y = \tan \frac{7x - 5}{3} = \tan t$.

$$\frac{dy}{dx} = \frac{dy}{dt} \cdot \frac{dt}{dx} = \frac{d}{dt} \tan t \cdot \frac{d}{dx} \frac{7x - 5}{3} = \sec^2 t \cdot \frac{7}{3} = \frac{7}{3} \sec^2 \frac{7x - 5}{3} .$$

問題 3.5.3 $t = 3x^2 - 7x + 5$ とおく. $f(x) = \ln(3x^2 - 7x + 5) = \ln t$ なのので,

$$\begin{aligned}f'(x) &= \frac{d}{dx} f(x) = \frac{d}{dx} \ln t = \frac{d}{dt} \ln t \cdot \frac{dt}{dx} = \frac{1}{t} \cdot \frac{d}{dx}(3x^2 - 7x + 5) = \frac{1}{3x^2 - 7x + 5} \cdot (6x - 7) \\ &= \frac{6x - 7}{3x^2 - 7x + 5} .\end{aligned}$$

問題 3.5.4 $t = \sin x$ とおく. $\psi(x) = \sin^3 x = t^3$ なのので,

$$\begin{aligned}\varphi'(x) &= \frac{d}{dx} \varphi(x) = \frac{d}{dx} t^3 = \frac{d}{dt} t^3 \cdot \frac{dt}{dx} = 3t^2 \cdot \frac{d}{dx} \sin x = 3(\sin x)^2 \cos x \\ &= 3 \sin^2 x \cos x .\end{aligned}$$

問題 3.5.5 変数 t を $t = \frac{7x + 8\pi}{3}$ とおく.

$$\varphi'(x) = \frac{d}{dx} \sin \frac{7x + 8\pi}{3} = \frac{d}{dx} \sin t = \frac{d}{dt} \sin t \cdot \frac{dt}{dx} = \cos t \cdot \frac{d}{dx} \frac{7x + 8\pi}{3} = \frac{7}{3} \cos \frac{7x + 8\pi}{3} .$$

$\frac{\pi}{2}$ における φ の微分係数は

$$\begin{aligned}\varphi'\left(\frac{\pi}{2}\right) &= \frac{7}{3} \cos \frac{7 \cdot \frac{\pi}{2} + 8\pi}{3} = \frac{7}{3} \cos \frac{23\pi}{6} = \frac{7}{3} \cos \left(\frac{23\pi}{6} - 4\pi\right) = \frac{7}{3} \cos \left(-\frac{\pi}{6}\right) = \frac{7}{3} \cos \frac{\pi}{6} \\ &= \frac{7\sqrt{3}}{6} .\end{aligned}$$