

§ 3.6 いくつかの関数の導関数

問題 3.6.1

$$\begin{aligned} f'(x) &= \frac{d}{dx} f(x) = \frac{d}{dx} (2^x \cos x) = \frac{d}{dx} 2^x \cdot \cos x + 2^x \frac{d}{dx} \cos x = 2^x \ln 2 \cos x + 2^x (-\sin x) \\ &= 2^x (\ln 2 \cos x - \sin x) . \end{aligned}$$

問題 3.6.2

$$\frac{dy}{dx} = \frac{d}{dx} \frac{\sin x}{3^x} = \frac{\frac{d}{dx} \sin x \cdot 3^x - \sin x \frac{d}{dx} 3^x}{(3^x)^2} = \frac{\cos x 3^x - \sin x 3^x \ln 3}{(3^x)^2} = \frac{\cos x - \ln 3 \sin x}{3^x} .$$

問題 3.6.3

$t = 5 - 3u$ とおく.

$$\frac{dv}{du} = \frac{d}{du} e^{5-3u} = \frac{d}{du} e^t = \frac{d}{dt} e^t \cdot \frac{dt}{du} = e^t \cdot \frac{d}{du} (5 - 3u) = e^{5-3u} \cdot (-3) = -3e^{5-3u} .$$

問題 3.6.4

$$\frac{d}{dx} \sqrt{x}^3 = \frac{d}{dx} x^{\frac{3}{2}} = \frac{3}{2} x^{\frac{1}{2}} = \frac{3}{2} \sqrt{x} .$$

問題 3.6.5

$$\frac{d}{dx} \frac{6}{\sqrt[3]{x}} = 6 \frac{d}{dx} x^{-\frac{1}{3}} = 6 \cdot \left(-\frac{1}{3} x^{-\frac{4}{3}} \right) = -\frac{2}{x^{\frac{4}{3}}} .$$

問題 3.6.6

$u = 3y^2 - 5y + 4$ とおく. $z = \sqrt{3y^2 - 5y + 4} = \sqrt{t} = t^{\frac{1}{2}}$ なので,

$$\frac{dz}{dy} = \frac{d}{dt} t^{\frac{1}{2}} \cdot \frac{dt}{dy} = \frac{1}{2} t^{-\frac{1}{2}} \cdot \frac{d}{dy} (3y^2 - 5y + 4) = \frac{1}{2\sqrt{t}} (6y - 5) = \frac{6y - 5}{2\sqrt{3y^2 - 5y + 4}} .$$

問題 3.6.7

変数 t を $t = 6x - 5$ とおく.

$$f'(x) = \frac{d}{dx} \sqrt{6x - 5}^3 = \frac{d}{dx} t^{\frac{3}{2}} = \frac{d}{dt} t^{\frac{3}{2}} \cdot \frac{dt}{dx} = \frac{3}{2} t^{\frac{1}{2}} \cdot \frac{d}{dx} (6x - 5) = \frac{3}{2} \sqrt{t} \cdot 6 = 9\sqrt{6x - 5} .$$

9 における f の微分係数は

$$f'(9) = 9\sqrt{6 \cdot 9 - 5} = 9\sqrt{49} = 63 .$$

問題 3.6.8

$t = \cos x$ とおく.

$$\begin{aligned} \frac{dy}{dx} &= \frac{d}{dx} \ln |\cos x| = \frac{d}{dx} \ln |t| = \frac{d}{dt} \ln |t| \cdot \frac{dt}{dx} = \frac{1}{t} \cdot \frac{d}{dx} \cos x = \frac{1}{\cos x} \cdot (-\sin x) = -\frac{\sin x}{\cos x} \\ &= -\tan x . \end{aligned}$$