

§ 3.7 逆三角関数の導関数

問題 3.7.1

$$\begin{aligned} \frac{dv}{du} &= \frac{d}{du} \frac{\tan^{-1}u}{u^2+1} = \frac{\frac{d}{du} \tan^{-1}u \cdot (u^2+1) - \tan^{-1}u \cdot \frac{d}{du}(u^2+1)}{(u^2+1)^2} = \frac{\frac{1}{u^2+1} \cdot (u^2+1) - \tan^{-1}u \cdot 2u}{(u^2+1)^2} \\ &= \frac{1 - 2u \tan^{-1}u}{(u^2+1)^2} . \end{aligned}$$

問題 3.7.2

$$\begin{aligned} g'(x) &= \frac{d}{dx} \{(1-x^2) \sin^{-1}x\} = \frac{d}{dx}(1-x^2) \cdot \sin^{-1}x + (1-x^2) \frac{d}{dx} \sin^{-1}x \\ &= -2x \sin^{-1}x + (1-x^2) \frac{1}{\sqrt{1-x^2}} \\ &= \sqrt{1-x^2} - 2x \sin^{-1}x . \end{aligned}$$

問題 3.7.3

$x = t^3$ とおく.

$$\frac{d}{dt} \tan^{-1}t^3 = \frac{d}{dt} \tan^{-1}x = \frac{d}{dx} \tan^{-1}x \cdot \frac{dx}{dt} = \frac{1}{1+x^2} \frac{d}{dt} t^3 = \frac{1}{1+(t^3)^2} 3t^2 = \frac{3t^2}{1+t^6} .$$

問題 3.7.4

$t = \cos^{-1}y$ とおく.

$$\begin{aligned} \frac{dz}{dy} &= \frac{d}{dy} \sqrt{\cos^{-1}y} = \frac{d}{dy} \sqrt{t} = \frac{d}{dt} t^{\frac{1}{2}} \cdot \frac{dt}{dy} = \frac{1}{2} t^{-\frac{1}{2}} \frac{d}{dy} \cos^{-1}y = \frac{1}{2\sqrt{\cos^{-1}t}} \cdot \left(-\frac{1}{\sqrt{1-y^2}} \right) \\ &= -\frac{1}{2\sqrt{(1-y^2)\cos^{-1}t}} . \end{aligned}$$