

§ 3.8 合成関数の微分法

問題 3.8.1

$$\frac{d}{dx} \sin \frac{3x-5}{4} = \cos \frac{3x-5}{4} \cdot \frac{d}{dx} \frac{3x-5}{4} = \cos \frac{3x-5}{4} \cdot \frac{3}{4} = \frac{3}{4} \cos \frac{3x-5}{4} .$$

問題 3.8.2

$$\frac{dy}{dx} = \frac{d}{dx} \ln(\sin x + 3) = \frac{1}{\sin x + 3} \frac{d}{dx} (\sin x + 3) = \frac{1}{\sin x + 3} \cos x = \frac{\cos x}{\sin x + 3} .$$

問題 3.8.3

$$\frac{d}{dx} \sin^4 x = \frac{d}{dx} (\sin x)^4 = 4(\sin x)^3 \frac{d}{dx} \sin x = 4 \sin^3 x \cos x .$$

問題 3.8.4

$$\frac{dy}{dx} = \frac{d}{dx} \tan^{-1} \sqrt{x} = \frac{1}{1 + \sqrt{x}^2} \frac{d}{dx} \sqrt{x} = \frac{1}{1+x} \frac{d}{dx} x^{\frac{1}{2}} = \frac{1}{1+x} \frac{1}{2} x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}(1+x)} .$$

問題 3.8.5

$$\frac{d}{dt} e^{3-2t} = e^{3-2t} \frac{d}{dx} (3-2t) = -2e^{3-2t} .$$

問題 3.8.6

$$\begin{aligned} \frac{dv}{du} &= \frac{d}{du} \sqrt{u^2 - 4u + 5} = \frac{d}{du} (u^2 - 4u + 5)^{\frac{1}{2}} \\ &= \frac{1}{2} (u^2 - 4u + 5)^{-\frac{1}{2}} \frac{d}{du} (u^2 - 4u + 5) = \frac{1}{2\sqrt{u^2 - 4u + 5}} (2u - 4) \\ &= \frac{u - 2}{\sqrt{u^2 - 4u + 5}} . \end{aligned}$$

問題 3.8.7

$$\begin{aligned} g'(x) &= \frac{d}{dx} \{e^x \cos(3x+1)\} = \frac{d}{dx} e^x \cdot \cos(3x+1) + e^x \frac{d}{dx} \cos(3x+1) \\ &= e^x \cos(3x+1) + e^x \{-\sin(3x+1)\} \frac{d}{dx} (3x+1) = e^x \cos(3x+1) + e^x \{-\sin(3x+1)\} 3 \\ &= e^x \{\cos(3x+1) - 3\sin(3x+1)\} . \end{aligned}$$

問題 3.8.8

$$\begin{aligned} \frac{dx}{dt} &= \frac{d}{dt} \frac{\sin t}{e^{3t-5}} = \frac{\frac{d}{dt} \sin t \cdot e^{3t-5} - \sin t \cdot \frac{d}{dt} e^{3t-5}}{(e^{3t-5})^2} \\ &= \frac{\cos t \cdot e^{3t-5} - \sin t \cdot e^{3t-5} \frac{d}{dt} (3t-5)}{(e^{3t-5})^2} = \frac{\cos t - \sin t \cdot 3}{e^{3t-5}} \\ &= \frac{\cos t - 3\sin t}{e^{3t-5}} . \end{aligned}$$