

§ 4.1 関数の極限

問題 4.1.1

$$\lim_{x \rightarrow \infty} \frac{\sqrt[3]{x^5}}{\sqrt[4]{x^9}} = \lim_{x \rightarrow \infty} \frac{x^{\frac{5}{3}}}{x^{\frac{9}{4}}} = \lim_{x \rightarrow \infty} x^{\frac{5}{3} - \frac{9}{4}} = \lim_{x \rightarrow \infty} x^{-\frac{7}{12}} = 0 .$$

$$\lim_{x \rightarrow \infty} \frac{\sqrt[4]{x^7}}{\sqrt[5]{x^8}} = \lim_{x \rightarrow \infty} \frac{x^{\frac{7}{4}}}{x^{\frac{8}{5}}} = \lim_{x \rightarrow \infty} x^{\frac{7}{4} - \frac{8}{5}} = \lim_{x \rightarrow \infty} x^{\frac{3}{20}} = \infty .$$

問題 4.1.2

$$\lim_{x \rightarrow \infty} \frac{5^x}{6^x} = \left(\frac{5}{6}\right)^x = 0 , \quad \lim_{x \rightarrow -\infty} \frac{5^x}{6^x} = \lim_{x \rightarrow -\infty} \left(\frac{5}{6}\right)^x = \infty .$$

問題 4.1.3

$$\lim_{x \rightarrow \infty} \frac{4^x}{3^x} = \lim_{x \rightarrow \infty} \left(\frac{4}{3}\right)^x = \infty , \quad \lim_{x \rightarrow -\infty} \frac{4^x}{3^x} = \lim_{x \rightarrow -\infty} \left(\frac{4}{3}\right)^x = 0 .$$