§4.7　級数

問題 4.7.1

$$\sum_{k=3}^{n}\left(\frac{6}{k}-\frac{6}{k+1}\right) = \frac{6}{3}-\frac{6}{4}+\frac{6}{4}-\frac{6}{5}+\cdots+\frac{6}{n-1}-\frac{6}{n}+\frac{6}{n}-\frac{6}{n+1} = 2-\frac{6}{n+1} \ .$$

$\lim_{n\to\infty}\frac{6}{n+1}=0$ なので，

$$\sum_{n=3}^{\infty}\left(\frac{6}{n}-\frac{6}{n+1}\right) = \lim_{n\to\infty}\sum_{k=3}^{n}\left(\frac{6}{k}-\frac{6}{k+1}\right) = \lim_{n\to\infty}\left(2-\frac{6}{n+1}\right) = 2 \ .$$

問題 4.7.2　$S_n=\sum_{k=2}^{n}\left\{6\left(\frac{2}{3}\right)^n\right\}$ とおく．

$$S_n = \frac{8}{3}+\frac{16}{9}+\frac{32}{27}+\cdots+6\left(\frac{2}{3}\right)^n \ ,$$

$$\frac{2}{3}S_n = \qquad \frac{16}{9}+\frac{32}{27}+\cdots+6\left(\frac{2}{3}\right)^n+6\left(\frac{2}{3}\right)^{n+1} \ ,$$

$\frac{1}{3}S_n=\frac{8}{3}-6\left(\frac{2}{3}\right)^{n+1}$ ，$S_n=8-18\left(\frac{2}{3}\right)^{n+1}$ ，$\sum_{k=2}^{n}\left\{6\left(\frac{2}{3}\right)^n\right\}=8-18\left(\frac{2}{3}\right)^{n+1}$ ．$\lim_{n\to\infty}\left(\frac{2}{3}\right)^{n+1}=0$ なので，

$$\sum_{n=1}^{\infty}\left\{10\left(\frac{3}{5}\right)^n\right\} = \lim_{n\to\infty}\sum_{k=2}^{n}\left\{6\left(\frac{2}{3}\right)^n\right\} = \lim_{n\to\infty}\left\{8-18\left(\frac{2}{3}\right)^{n+1}\right\} = 8-18\cdot 0 = 8 \ .$$

問題 4.7.3

$$\sum_{k=0}^{n}\left\{3\left(\frac{2}{5}\right)^k\right\} = 3\frac{1-\left(\frac{2}{5}\right)^{n+1}}{1-\frac{2}{5}} = 5\left\{1-\left(\frac{2}{5}\right)^{n+1}\right\} \ .$$

$\lim_{n\to\infty}\left(\frac{2}{5}\right)^{n+1}=0$ なので，

$$\sum_{n=0}^{\infty}\left\{3\left(\frac{2}{5}\right)^n\right\} = \lim_{n\to\infty}\sum_{k=0}^{n}\left\{3\left(\frac{9}{7}\right)^k\right\} = \lim_{n\to\infty}\left[3\left\{1-\left(\frac{2}{5}\right)^{n+1}\right\}\right] = 3 \ .$$

問題 4.7.4　公比 $-\frac{7}{6}$ の等比数列 $\left\{\left(-\frac{7}{6}\right)^n\right\}_{n\geq 0}$ は発散するので，等比級数 $\sum_{n=0}^{\infty}\left(-\frac{7}{6}\right)^n$ は発散する．

問題 4.7.5　$\lim_{n\to\infty}\frac{8}{n^2+3}=0$ なので，$\lim_{n\to\infty}\left(\frac{8}{n^2+3}-7\right)=0-7=-7$ ．よって $\lim_{n\to\infty}\left(\frac{8}{n^2+3}\right)\neq 0$ なので，級数 $\sum_{n=0}^{\infty}\left(\frac{8}{n^2+3}-7\right)$ は発散する．

問題 4.7.6

$$\sum_{k=0}^{n}\frac{6}{4^k} = \sum_{k=0}^{n}\left\{6\left(\frac{1}{4}\right)^k\right\} = 6\sum_{k=0}^{n}\left(\frac{1}{4}\right)^k = 6\frac{1-\left(\frac{1}{4}\right)^{n+1}}{1-\frac{1}{4}} = 6\frac{1-\left(\frac{1}{4}\right)^{n+1}}{\frac{3}{4}} = 8\left(1-\frac{1}{4^{n+1}}\right) \ .$$

$\lim_{n\to\infty}\frac{1}{4^{n+1}}=0$ なので，

$$\sum_{n=0}^{\infty}\frac{6}{4^n} = \lim_{n\to\infty}\sum_{k=0}^{n}\frac{6}{4^k} = \lim_{n\to\infty}\left\{8\left(1-\frac{1}{4^{n+1}}\right)\right\} = 8 \ .$$

問題 4.7.7

$$\sum_{n=1}^{\infty}\left\{3\left(\frac{2}{5}\right)^n\right\} = \frac{6}{5}+\frac{6}{5}\cdot\frac{2}{5}+\frac{6}{5}\left(\frac{2}{5}\right)^2+\frac{6}{5}\left(\frac{2}{5}\right)^3+\cdots = \frac{\frac{6}{5}}{1-\frac{2}{5}} = \frac{6}{3} = 2 \ .$$