§6.6　不定積分の公式

問題 6.6.1

$$\frac{d}{dx}(-\cos x) = -\frac{d}{dx}\cos x = -(-\sin x) = \sin x \ .$$

従って，積分定数を C とおくと，

$$\int \sin x\, dx = \int \Big\{ \frac{d}{dx}(-\cos x) \Big\}\, dx = -\cos x + C \ .$$

問題 6.6.2

$$\frac{d}{dx}\Big(\frac{1}{a}\tan^{-1}\frac{x}{a}\Big) = \frac{1}{a}\frac{1}{1+\Big(\frac{x}{a}\Big)^2}\frac{1}{a} = \frac{1}{x^2+a^2} \ .$$

従って，積分定数を C とおくと，

$$\int \frac{1}{x^2+a^2}\, dx = \int \Big[\frac{d}{dx}\Big(\frac{1}{a}\tan^{-1}\frac{x}{a}\Big) \Big]\, dx = \frac{1}{a}\tan^{-1}\frac{x}{a} + C \ .$$

問題 6.6.3

(1) 積分定数を C とおくと，

$$\int \frac{1}{x^4}\, dx = \int x^{-4} dx = \frac{1}{-3}x^{-3} + C = -\frac{1}{3x^3} + C \ .$$

(2) 積分定数を C とおくと，

$$\int \frac{1}{\sqrt{t}}\, dt = \int x^{-\frac{1}{2}} dx = \frac{1}{\frac{1}{2}}x^{\frac{1}{2}} + C = 2\sqrt{x} + C \ .$$

問題 6.6.4

$$\int e^{\frac{x}{4}}\, dx = \int \big(e^{\frac{1}{4}}\big)^x dx = \frac{\big(e^{\frac{1}{4}}\big)^x}{\ln e^{\frac{1}{4}}} + C = 4e^{\frac{x}{4}} + C \quad （C \text{ は積分定数}）.$$

問題 6.6.5

(1) 積分定数を C とおくと，

$$\int \frac{1}{\sqrt{7-y^2}}\, dy = \sin^{-1}\frac{y}{\sqrt{7}} + C \ .$$

(2) 積分定数を C とおくと，

$$\int \frac{1}{u^2+\frac{1}{9}}\, du = \frac{1}{\frac{1}{3}}\tan^{-1}\frac{u}{\frac{1}{3}} + C = 3\tan^{-1}(3u) + C \ .$$