§ 6.8　定積分と不定積分

問題 6.8.1　積分定数を C とおくと，

$$\int(2y^2-5y+3)\,dy = 2\cdot\frac{1}{3}y^3-5\cdot\frac{1}{2}y^2+3y+C = \frac{2}{3}y^3-\frac{5}{2}y^2+3y+C\ .$$

よって，

$$\int_{-2}^{4}(2y^2-5y+3)\,dy = \left[\frac{2}{3}y^3-\frac{5}{2}y^2+3y\right]_{-2}^{4} = \frac{128}{3}-40+12-\left(-\frac{16}{3}8-10-6\right) = 34\ .$$

問題 6.8.2　積分定数を C とおくと，

$$\int\frac{4\cos\theta-3}{5}\,d\theta = \frac{4\int\cos\theta-\int 3\,d\theta}{5} = \frac{4\sin\theta-3\theta}{5}+C\ .$$

よって，

$$\int_{\frac{\pi}{2}}^{-\frac{\pi}{3}}\frac{4\cos\theta-3}{5}\,d\theta = \left[\frac{4\sin\theta-3\theta}{5}\right]_{\frac{\pi}{2}}^{-\frac{\pi}{3}} = \frac{1}{5}\left\{4\sin\left(-\frac{\pi}{3}\right)+3\left(-\frac{\pi}{3}\right)-4\sin\frac{\pi}{2}-3\frac{\pi}{2}\right\}$$

$$=\frac{\pi}{2}-\frac{4}{5}-\frac{2\sqrt{3}}{5}\ .$$

問題 6.8.3

(1)　積分定数を C とおくと，

$$\int\frac{3}{\sqrt{4-9x^2}}\,dx = \int\frac{3}{\sqrt{9\left(\frac{4}{9}-x^2\right)}}\,dx = \int\frac{1}{\sqrt{\frac{4}{9}-x^2}}\,dx = \sin^{-1}\frac{x}{\frac{2}{3}}+C = \sin^{-1}\frac{3x}{2}+C\ .$$

よって，

$$\int_{-\frac{1}{3}}^{\frac{1}{2}}\frac{3}{\sqrt{4-9x^2}}\,dx = \left[\sin^{-1}\frac{3x}{2}\right]_{-\frac{1}{3}}^{\frac{1}{2}} = \sin^{-1}\frac{3}{4}-\sin^{-1}\left(-\frac{1}{2}\right) = \frac{\pi}{6}+\sin^{-1}\frac{3}{4}\ .$$

(2)　積分定数を C とおくと，

$$\int\frac{5t+4}{t^2}\,dt = \int\left(5\frac{1}{t}+4\frac{1}{t^2}\right)dt = 5\int\frac{1}{t}\,dt+4\int t^{-2}dt = 5\ln|t|+4(-t^{-1})+C$$

$$=5\ln|t|-\frac{4}{t}+C\ .$$

よって，

$$\int_{\frac{1}{3}}^{1}\frac{5t+4}{t^2}\,dt = \left[5\ln|t|-\frac{4}{t}\right]_{\frac{1}{3}}^{1} = 5\ln 1-4-5\ln\frac{1}{3}+12 = 8+5\ln 3\ .$$