

§ 6.9 定積分の性質

問題 6.9.1

$$(1) \int_2^0 (5x^3 - 3e^x) dx = 5 \int_2^0 x^3 dx - 3 \int_2^0 e^x dx = 5 \left[\frac{1}{4} x^4 \right]_2^0 - 3 [e^x]_2^0 = -20 - 3(1 - e^2) = 3e^2 - 23 .$$

$$(2) \int_0^\pi \frac{x - 3 \sin x}{5} dx = \frac{1}{5} \left(\int_0^\pi x dx - 3 \int_0^\pi \sin x dx \right) = \frac{1}{5} \left\{ \left[\frac{1}{2} x^2 \right]_0^\pi - 3 [-\cos x]_0^\pi \right\} = \frac{1}{5} \left(\frac{\pi^2}{2} - 6 \right) \\ = \frac{\pi^2 - 12}{10} .$$

問題 6.9.2

$x \neq 2$ のとき $g(x) = \frac{2}{x^2 + 3}$ なので,

$$\int_0^3 g(x) dx = \int_0^3 \frac{2}{x^2 + 3} dx = \left[\frac{2}{\sqrt{3}} \tan^{-1} \frac{x}{\sqrt{3}} \right]_0^3 = \frac{2}{\sqrt{3}} \cdot \frac{\pi}{3} = \frac{2\pi}{3\sqrt{3}} .$$

問題 6.9.3

$0 \leq x \leq \frac{3}{2}$ である各実数 x について, $x \neq 0$, $x \neq \frac{3}{2}$ のとき $f(x) = \frac{6}{\sqrt{3-x^2}}$

なので,

$$\int_0^{\frac{3}{2}} f(x) dx = \int_0^{\frac{3}{2}} \frac{6}{\sqrt{3-x^2}} dx = 6 \left[\sin^{-1} \frac{x}{\sqrt{3}} \right]_0^{\frac{3}{2}} = 6 \left(\sin^{-1} \frac{\sqrt{3}}{2} - \sin^{-1} 0 \right) = 6 \cdot \frac{\pi}{3} = 2\pi .$$

問題 6.9.4

$$\int_0^\pi \varphi(x) dx = \int_0^2 \varphi(x) dx + \int_2^\pi \varphi(x) dx .$$

$$\int_0^2 \varphi(x) dx = \int_0^2 \sin x dx = [-\cos x]_0^2 = -\cos 2 + \cos 0 = 1 - \cos 2 .$$

$$\int_2^\pi \varphi(x) dx = \int_2^\pi \cos 2 dx = (\cos 2) \int_2^\pi 1 dx = (\cos 2) [x]_2^\pi = (\pi - 2) \cos 2 .$$

よって

$$\int_0^\pi \varphi(x) dx = \int_0^2 \varphi(x) dx + \int_2^\pi \varphi(x) dx = 1 - \cos 2 + (\pi - 2) \cos 2 = 1 + (\pi - 3) \cos 2 .$$