§7.1　不定積分の置換積分法

$\boxed{\text{問題 7.1.1}}$　変数 y を　$y = 5x - 4$　とおく．　$\dfrac{dy}{dx} = \dfrac{d}{dx}(5x-4) = 5$　なので　$dy = 5\,dx$ ，$dx = \dfrac{1}{5}\,dy$ ，よって

$$\sin(5x-4)\,dx = \sin y\,\frac{1}{5}\,dy = \frac{1}{5}\sin y\,dy\ .$$

積分定数を C とおくと，

$$\int\sin(5x-4)\,dx = \int\frac{1}{5}\sin y\,dy = \frac{1}{5}\int\sin y\,dy = \frac{1}{5}(-\cos y) + C = -\frac{1}{5}\cos(5x-4) + C\ .$$

$\boxed{\text{問題 7.1.2}}$　変数 z を　$z = 4y + 5$　とおく．　$\dfrac{dz}{dy} = \dfrac{d}{dy}(4y+5) = 4$　なので，　$dz = 4\,dy$ ，$dy = \dfrac{1}{4}\,dz$ ．　よって

$$\frac{9}{(4y+5)^3}\,dy = \frac{9}{z^3}\,\frac{1}{4}\,dz = \frac{9}{4}\,\frac{1}{z^3}\,dz\ .$$

積分定数を C とおくと，

$$\int\frac{9}{(4y+5)^3}\,dy = \int\frac{9}{4}\frac{1}{z^3}\,dz = \frac{9}{4}\int z^{-3}\,dz = \frac{9}{4}\,\frac{1}{-2}z^{-2} + C = -\frac{9}{8(4y+5)^2} + C\ .$$

$\boxed{\text{問題 7.1.3}}$

(1)　変数 v を　$v = 4u + 5$　とおく．　$\dfrac{dv}{du} = 4$　なので，　$dv = 4\,du$ ，　$du = \dfrac{1}{4}\,dv$ ．　よって

$$\frac{9}{4u+5}\,du = \frac{9}{v}\,\frac{1}{4}\,dv = \frac{9}{4}\,\frac{1}{v}\,dv\ .$$

積分定数を C とおくと，

$$\int\frac{9}{4u+5}\,du = \int\frac{9}{4}\,\frac{1}{v}\,dv = \frac{9}{4}\int\frac{1}{v}\,dv = \frac{9}{4}\ln|v| + C = \frac{9}{4}\ln|4u+5| + C\ .$$

(2)　変数 y を　$y = 6x + 5$　とおく．　$\dfrac{dy}{dx} = 6$　なので，　$dy = 6\,dx$ ，　$dx = \dfrac{1}{6}\,dy$ ．　よって

$$\sqrt{6x+5}\,dx = \sqrt{y}\,\frac{1}{6}\,dy = \frac{1}{6}\sqrt{y}\,dy\ .$$

積分定数を C とおくと，

$$\int\sqrt{6x+5}\,dx = \int\frac{1}{6}\sqrt{y}\,dy = \frac{1}{6}\int y^{\frac{1}{2}}\,dy = \frac{1}{6}\,\frac{2}{3}y^{\frac{3}{2}} + C = \frac{1}{9}\sqrt{6x+5}^{\,3} + C\ .$$

$\boxed{\text{問題 7.1.4}}$

(1)　変数 y を　$y = \dfrac{3x-7}{5}$　とおく．　$\dfrac{dy}{dx} = \dfrac{3}{5}$　なので　$dx = \dfrac{5}{3}\,dy$ ．　よって

$$\sin\frac{3x-7}{5}\,dx = \sin y\,\frac{5}{3}\,dy = \frac{5}{3}\sin y\,dy\ .$$

積分定数を C とおくと，

$$\int\sin\frac{3x-7}{5}\,dx = \int\frac{5}{3}\sin y\,dy = \frac{5}{3}\int\sin y\,dy = \frac{5}{3}(-\cos y) + C = -\frac{5}{3}\cos\frac{3x-7}{5} + C\ .$$

(2)　変数 x を　$x = 2t + 3$　とおく．　$\dfrac{dx}{dt} = 2$　なので　$dt = \dfrac{1}{2}\,dx$ ．　よって

$$e^{2t+3}dt = e^x\,\frac{1}{2}\,dx = \frac{1}{2}e^x dx\ .$$

積分定数を C とおくと，

$$\int e^{2t+3}dt = \int e^x\,\frac{1}{2}\,dx = \frac{1}{2}\int e^x dx = \frac{1}{2}e^x + C = \frac{1}{2}e^{2t+3} + C\ .$$

$\boxed{\text{問題 7.1.5}}$　変数 y を　$y = x^2 + 3$　とおく．　$\dfrac{dy}{dx} = 2x$　なので　$x\,dx = \dfrac{1}{2}\,dy$ ．　よって

$$\frac{x}{\sqrt{x^2+3}}\,dx = \frac{1}{\sqrt{x^2+3}}\,x\,dx = \frac{1}{\sqrt{y}}\,\frac{1}{2}\,dy = \frac{1}{2}\,\frac{1}{\sqrt{y}}\,dy\ .$$

積分定数を C とおくと，

$$\int\frac{x}{\sqrt{x^2+3}}\,dx = \int\frac{1}{2}\,\frac{1}{\sqrt{y}}\,dy = \frac{1}{2}\int y^{-\frac{1}{2}}\,dy = \frac{1}{2}\,2y^{\frac{1}{2}} + C = \sqrt{y} + C = \sqrt{x^2+3} + C\ .$$

$\boxed{\text{問題 7.1.6}}$　変数 x を　$x = \cos t$　とおく．　$\dfrac{dx}{dt} = -\sin t$　なので　$\sin t\,dt = -dx$ ．　よって

$$(\cos^2 t + 3)\sin t\,dt = (x^2+3)(-dx) = (-x^2-3)\,dx\ .$$

積分定数を C とおくと，

$$\int(\cos^2 t + 3)\sin t\,dt = \int(-x^2-3)\,dx = -\frac{1}{3}x^3 - 3x + C = -\frac{1}{3}\cos^3 t - 3\cos t + C\ .$$

$\boxed{\text{問題 7.1.7}}$　変数 z を　$z = \dfrac{y}{3}$　とおく．　$\dfrac{dz}{dy} = \dfrac{1}{3}$　なので　$dy = 3\,dz$ ．　よって

$$\tan\frac{y}{3}\,dy = (\tan z)\,3\,dz = 3\tan z\,dz\ .$$

積分定数を C とおくと，

$$\int\tan\frac{y}{3}\,dy = \int 3\tan z\,dz = 3\int\tan z\,dz = 3(-\ln|\cos z|) + C = -3\ln\left|\cos\frac{y}{3}\right| + C\ .$$

$\boxed{\text{問題 7.1.8}}$　変数 y を　$y = x^3 + 2$　とおく．　$\dfrac{dy}{dx} = 3x^2$　なので　$x^2\,dx = \dfrac{1}{3}\,dy$ ．　積分定数を C とおくと，

$$\int\frac{x^2}{x^3+2}\,dx = \int\frac{1}{x^3+2}\,x^2 dx = \int\frac{1}{y}\,\frac{1}{3}\,dy = \frac{1}{3}\int\frac{1}{y}\,dy = \frac{1}{3}\ln|y| + C = \frac{1}{3}\ln|x^3+2| + C\ .$$