§7.3　部分積分法

問題 7.3.1　積分定数を C_1 とおくと,

$$\int x\sin x\,dx = x(-\cos x) - \int(-\cos x)\,dx = -x\cos x + \int \cos x\,dx = -x\cos x + \sin x + C_1$$
$$= \sin x - x\cos x + C_1 \ .$$

積分定数を C_2 とおくと,

$$\int x^2\cos x\,dx = x^2\sin x + \int 2x\sin x\,dx = x^2\sin x + 2\int x\sin x\,dx$$
$$= x^2\sin x - 2(\sin x - x\cos x) + C_2 = (x^2-2)\sin x + 2x\cos x + C_2 \ .$$

問題 7.3.2　積分定数を C_1 とおくと,

$$\int xe^x dx \ = \ xe^x - \int e^x \ = \ xe^x - e^x + C_1 \ = \ e^x(x-1) + C_1 \ .$$

積分定数を C_2 とおくと,

$$\int x^2e^x dx = x^2e^x - \int 2xe^x dx = x^2e^x - 2\int xe^x dx = x^2e^x - 2e^x(x-1) + C_2$$
$$= e^x(x^2-2x+2) + C_2 \ .$$

問題 7.3.3　$v = 4u+7$ とおく. $\dfrac{dv}{du} = 4$ より $du = \dfrac{1}{4}dv$. 積分定数を C_0, C とおくと,

$$\int \ln(4u+5)\,du = \int (\ln v)\frac{1}{4}\,dv = \frac{v}{4}(\ln v - 1) + C_0 = \frac{4u+7}{4}\{\ln(4u+7)-1\} + C_0$$
$$= \left(u+\frac{7}{4}\right)\ln(4u+7) - u + C \ .$$

問題 7.3.4　積分定数を C とおくと,

$$\int x^3\ln x\,dx = \frac{1}{4}x^4\ln x - \frac{1}{4}\int x^4\frac{1}{x}\,dx = \frac{x^4}{4}\ln x - \frac{1}{4}\int x^3 dx = \frac{x^4}{4}\ln x - \frac{1}{4}\frac{x^4}{4} + C$$
$$= \frac{x^4}{4}\left(\ln x - \frac{1}{4}\right) + C \ .$$

問題 7.3.5　積分定数を略すと,

$$\int \cos 5x\,dx \ = \ \frac{1}{5}\sin 5x \ , \qquad \int \sin 5x\,dx \ = \ -\frac{1}{5}\cos 5x \ .$$

積分定数を C とおくと,

$$\int x\cos 5x\,dx = x\cdot\frac{1}{5}\sin 5x - \frac{1}{5}\int \sin 5x\,dx = \frac{x}{5}\sin 5x - \frac{1}{5}\cdot\left(-\frac{1}{5}\cos 5x\right) + C$$
$$= \frac{x}{5}\sin 5x + \frac{1}{25}\cos 5x + C \ .$$

問題 7.3.6　積分定数を省略すると $\int e^{\frac{x}{3}}dx = 3e^{\frac{x}{3}}$. 積分定数を C とおくと,

$$\int xe^{\frac{x}{3}}\,dx \ = \ 3xe^{\frac{x}{3}} - 3\int e^{\frac{x}{3}}\,dx \ = \ 3xe^{\frac{x}{3}} - 9e^{\frac{x}{3}} + C \ = \ 3e^{\frac{x}{3}}(x-3) + C \ .$$

問題 7.3.7　積分定数を C とおくと,

$$\int t\sin t\,dt \ = \ -t\cos t + \int \cos t\,dt \ = \ \sin t - t\cos t + C \ .$$

よって

$$\int_{\frac{\pi}{3}}^{\pi} t\sin t\,dt = \Big[\sin t - t\cos t\Big]_{\frac{\pi}{3}}^{\pi} = \sin\pi - \pi\cos\pi - \left(\sin\frac{\pi}{3} - \frac{\pi}{3}\cos\frac{\pi}{3}\right) = \pi - \frac{\sqrt{3}}{2} + \frac{\pi}{6}$$
$$= \frac{7\pi}{6} - \frac{\sqrt{3}}{2} \ .$$