

§ 7.4 有理関数の積分法

問題 7.4

(1) $3x = \frac{3}{2}(2x+5) - \frac{15}{2}$ なので, $\frac{3x}{2x+5} = \frac{3}{2} - \frac{15}{2} \frac{1}{2x+5}$. 変数 y を $y = 2x+5$ とおく.

$\frac{dy}{dx} = 2$ なので $dx = \frac{1}{2}dy$. 積分定数を C とおくと,

$$\begin{aligned}\int \frac{3x}{2x+5} dx &= \int \frac{3}{2} dx - \int \frac{15}{2} \frac{1}{2x+5} dx = \frac{3}{2}x - \frac{15}{2} \int \frac{1}{y} \frac{1}{2} dx = \frac{3}{2}x - \frac{15}{4} \ln|y| + C \\ &= \frac{3}{2}x - \frac{15}{4} \ln|2x+5| + C.\end{aligned}$$

(2) $2x^2 + 5x = (x+3)(2x-1) + 3$ なので $\frac{2x^2+5x}{2x-1} = x+3 + \frac{3}{2x-1}$. 変数 y を $y = 2x-1$

とおく. $\frac{dy}{dx} = 2$ なので $dx = \frac{1}{2}dy$. 積分定数を C とおくと,

$$\begin{aligned}\int \frac{2x^2+5x}{2x-1} dx &= \int (x+3) dx + \int \frac{3}{2x-1} dx = \frac{1}{2}x^2 + 3x + 3 \int \frac{1}{y} \frac{1}{2} dy = \frac{1}{2}x^2 + 3x + \frac{3}{2} \ln|y| + C \\ &= \frac{1}{2}x^2 + 3x + \frac{3}{2} \ln|2x-1| + C.\end{aligned}$$