§1.6　根号

問題 1.6.1

(1)
$$\sqrt{7}\sqrt{21}+\frac{\sqrt{60}}{\sqrt{5}}=\sqrt{3\cdot 7^2}+\sqrt{12}=7\sqrt{3}+2\sqrt{3}=9\sqrt{3}\ .$$

(2)
$$\sqrt{6}\sqrt{15}-\frac{\sqrt{120}}{\sqrt{3}}=\sqrt{90}-\sqrt{40}=3\sqrt{10}-2\sqrt{10}=\sqrt{10}\ .$$

問題 1.6.2

$$\left(\frac{\sqrt{7}-\sqrt{3}}{\sqrt{2}}\right)^2=\frac{\sqrt{7}^2-2\sqrt{7}\sqrt{3}+\sqrt{3}^2}{\sqrt{2}^2}=\frac{7-2\sqrt{21}+3}{2}=\frac{10-2\sqrt{21}}{2}=5-\sqrt{21}\ .$$

問題 1.6.3

$$\big(4-\sqrt{5}\big)\big(3+2\sqrt{5}\big)=12+8\sqrt{5}-3\sqrt{5}-10=2+5\sqrt{5}\ .$$

問題 1.6.4

$$\big(4-\sqrt{7}\big)^3=4^3-3\cdot 4^2\sqrt{7}+3\cdot 4\sqrt{7}^2-\sqrt{7}^3=64-48\sqrt{7}+84-7\sqrt{7}=148-55\sqrt{7}\ .$$

問題 1.6.5

$$\frac{\sqrt{60}}{\sqrt{35}}=\sqrt{\frac{60}{35}}=\sqrt{\frac{12}{7}}=\frac{\sqrt{12}}{\sqrt{7}}=\frac{2\sqrt{3}\,\sqrt{7}}{\sqrt{7}^2}=\frac{2}{7}\sqrt{21}\ .$$

問題 1.6.6

$$\frac{\sqrt{96}-4\sqrt{5}}{\sqrt{2}}=\frac{\sqrt{96}}{\sqrt{2}}-\frac{4\sqrt{5}}{\sqrt{2}}=\sqrt{\frac{96}{2}}-\frac{4\sqrt{5}\sqrt{2}}{\sqrt{2}^2}=\sqrt{48}-\frac{4\sqrt{10}}{2}=4\sqrt{3}-2\sqrt{10}\ .$$

問題 1.6.7

(1)
$$\frac{12}{\sqrt{7}-\sqrt{3}}=\frac{12\big(\sqrt{7}+\sqrt{3}\big)}{\big(\sqrt{7}-\sqrt{3}\big)\big(\sqrt{7}+\sqrt{3}\big)}=\frac{12\big(\sqrt{7}+\sqrt{3}\big)}{\sqrt{7}^2-\sqrt{3}^2}=\frac{12\big(\sqrt{7}+\sqrt{3}\big)}{7-3}=3\big(\sqrt{7}+\sqrt{3}\big)\ .$$

(2)
$$\frac{3}{2+\sqrt{5}}=\frac{3\big(2-\sqrt{5}\,\big)}{\big(2+\sqrt{5}\,\big)\big(2-\sqrt{5}\,\big)}=\frac{3\big(2-\sqrt{5}\,\big)}{2^2-\sqrt{5}^2}=\frac{3\big(2-\sqrt{5}\,\big)}{4-5}=-3\big(2-\sqrt{5}\,\big)=3\sqrt{5}-6\ .$$

問題 1.6.8

(1)
$$\frac{4+\sqrt{5}}{3+\sqrt{5}}=\frac{\big(4+\sqrt{5}\big)\big(3-\sqrt{5}\big)}{\big(3+\sqrt{5}\big)\big(3-\sqrt{5}\big)}=\frac{12-4\sqrt{5}+3\sqrt{5}-\sqrt{5}^2}{3^2-\sqrt{5}^2}=\frac{7-\sqrt{5}}{4}\ .$$

(2)
$$\begin{aligned}\frac{3\sqrt{2}-\sqrt{5}}{\sqrt{5}-\sqrt{2}}&=\frac{\big(3\sqrt{2}-\sqrt{5}\big)\big(\sqrt{5}+\sqrt{2}\big)}{\big(\sqrt{5}-\sqrt{2}\big)\big(\sqrt{5}+\sqrt{2}\big)}\\&=\frac{3\sqrt{2}\sqrt{5}+3\sqrt{2}^2-\sqrt{5}^2-\sqrt{5}\sqrt{2}}{\sqrt{5}^2-\sqrt{3}^2}=\frac{3\sqrt{10}+6-5-\sqrt{10}}{3}\\&=\frac{1+2\sqrt{10}}{3}\ .\end{aligned}$$