

§ 10.10 逆三角関数

問題 10.10.1

- (1) $\sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$ なので $\sin^{-1} \frac{\sqrt{3}}{2} = \sin^{-1} \left(\sin \frac{\pi}{3} \right) = \frac{\pi}{3}$.
- (2) $\sin 0 = 0$ なので $\sin^{-1} 0 = \sin^{-1}(\sin 0) = 0$.
- (3) $\sin 1 = \frac{\pi}{2}$ なので $\sin^{-1}(-1) = -\sin^{-1} 1 = -\sin^{-1} \left(\sin \frac{\pi}{2} \right) = -\frac{\pi}{2}$.
- (4) $\tan \frac{\pi}{4} = 1$ なので $\tan^{-1} 1 = \tan^{-1} \left(\tan \frac{\pi}{4} \right) = \frac{\pi}{4}$.
- (5) $\tan 0 = 0$ なので $\tan^{-1} 0 = \tan^{-1}(\tan 0) = 0$.
- (6) $\tan \frac{\pi}{6} = \frac{1}{\sqrt{3}}$ なので $\tan^{-1} \left(-\frac{1}{\sqrt{3}} \right) = -\tan^{-1} \frac{1}{\sqrt{3}} = -\tan^{-1} \left(\tan \frac{\pi}{6} \right) = -\frac{\pi}{6}$.

問題 10.10.2 $x = \sin^{-1} \frac{\sqrt{5}}{3}$ とおく. $\sin x = \sin \left(\sin^{-1} \frac{\sqrt{5}}{3} \right) = \frac{\sqrt{5}}{3}$ なので,

$$\cos^2 x = 1 - \sin^2 x = 1 - \left(\frac{\sqrt{5}}{3} \right)^2 = 1 - \frac{5}{9} = \frac{4}{9} .$$

$-\frac{\pi}{2} \leq \sin^{-1} \frac{\sqrt{5}}{3} \leq \frac{\pi}{2}$ なので $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$, 従つて $\cos x \geq 0$ なので, $\cos x = \sqrt{\frac{4}{9}} = \frac{2}{3}$.

故に $\cos \left(\sin^{-1} \frac{\sqrt{5}}{3} \right) = \cos x = \frac{2}{3}$.

問題 10.10.3 $x = \tan^{-1} \left(-\frac{5}{2} \right)$ とおく. $\tan x = \tan \left\{ \tan^{-1} \left(-\frac{5}{2} \right) \right\} = -\frac{5}{2}$ なので,

$$\cos^2 x = \frac{1}{1 + \tan^2 x} = \frac{1}{1 + \left(-\frac{5}{2} \right)^2} = \frac{4}{29} .$$

$-\frac{\pi}{2} < \tan^{-1} \left(-\frac{5}{2} \right) < \frac{\pi}{2}$ なので $-\frac{\pi}{2} < x < \frac{\pi}{2}$, 従つて $\cos x > 0$ なので, $\cos x = \sqrt{\frac{4}{29}} = \frac{2}{\sqrt{29}}$.

故に $\cos \left\{ \tan^{-1} \left(-\frac{5}{2} \right) \right\} = \cos x = \frac{2}{\sqrt{29}}$.

問題 10.10.4 $-\frac{\pi}{2} \leq 2\pi - 7 \leq \frac{\pi}{2}$ なので,

$$\sin^{-1} \{ \sin(-7) \} = \sin^{-1} \{ \sin(2\pi - 7) \} = 2\pi - 7 .$$

問題 10.10.5 $-\frac{\pi}{2} \leq 13 - 4\pi \leq \frac{\pi}{2}$ なので,

$$\sin^{-1}(\sin 13) = \sin^{-1} \{ \sin(13 - 4\pi) \} = 13 - 4\pi .$$

問題 10.10.6 $-\frac{\pi}{2} < \pi - 3 < \frac{\pi}{2}$ なので,

$$\tan^{-1} \{ \tan(-3) \} = \tan^{-1} \{ \tan(\pi - 3) \} = \pi - 3 .$$

問題 10.10.7 $-\frac{\pi}{2} < 9 - 3\pi < \frac{\pi}{2}$ なので,

$$\tan^{-1}(\tan 9) = \tan^{-1} \{ \tan(9 - 3\pi) \} = 9 - 3\pi .$$