

§ 10.8 三角関数の相互関係

問題 10.8.1

$$\cos^2 x = 1 - \sin^2 x = 1 - \left(-\frac{2}{3}\right)^2 = \frac{5}{9} .$$

$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ より $\cos x \geq 0$ なので, $\cos x = \sqrt{\frac{5}{9}} = \frac{\sqrt{5}}{3}$. 更に

$$\tan x = \frac{\sin x}{\cos x} = \frac{-\frac{2}{3}}{\frac{\sqrt{5}}{3}} = -\frac{2}{\sqrt{5}} .$$

問題 10.8.2

$$\cos^2 x = \frac{1}{1 + \tan^2 x} = \frac{1}{1 + \left(\frac{5}{3}\right)^2} = \frac{9}{34} .$$

$\pi \leq x \leq 2\pi$ かつ $\tan x > 0$ より $\pi < x < \frac{3\pi}{2}$ なので, $\cos x < 0$. 従って

$\cos x = -\sqrt{\frac{9}{34}} = -\frac{3}{\sqrt{34}}$. 更に,

$$\sin x = \tan x \cos x = \frac{5}{3} \cdot \left(-\frac{3}{\sqrt{34}}\right) = -\frac{5}{\sqrt{34}} .$$

問題 10.8.3

$\sin^2 t + \cos^2 t = 1$ なので, $\sin^2 t + \left(\frac{3}{4} \sin t\right)^2 = 1$, $\frac{25}{16} \sin^2 t = 1$, $\sin^2 t = \frac{16}{25}$;

$\frac{\pi}{2} \leq t \leq \frac{3\pi}{2}$ より $\cos t \leq 0$ なので $\sin t = \frac{4}{3} \cos t \leq 0$, よって $\sin t = -\sqrt{\frac{16}{25}} = -\frac{4}{5}$.