

§ 4.6 2次式の平方完成

問題 4.6.1

$$\begin{aligned}3x^2 - 5x + 1 &= 3\left(x^2 - \frac{5}{3}x\right) + 1 = 3\left\{x^2 - 2 \cdot \frac{5}{6}x + \left(\frac{5}{6}\right)^2 - \frac{5^2}{6^2}\right\} + 1 = 3\left(x - \frac{5}{6}\right)^2 - \frac{25}{12} + \frac{12}{12} \\ &= 3\left(x - \frac{5}{6}\right)^2 - \frac{13}{12} .\end{aligned}$$

問題 4.6.2

$$\begin{aligned}-5x^2 - 6x - 3 &= -5\left(x^2 + \frac{6}{5}x\right) - 3 = -5\left\{x^2 + 2 \cdot \frac{3}{5}x + \left(\frac{3}{5}\right)^2 - \frac{3^2}{5^2}\right\} - 3 = -5\left(x + \frac{3}{5}\right)^2 + \frac{9}{5} - \frac{15}{5} \\ &= -5\left(x + \frac{3}{5}\right)^2 - \frac{6}{5} .\end{aligned}$$

問題 4.6.3

$$\begin{aligned}(1) \quad \frac{5}{2}x^2 - 4x + 3 &= \frac{5}{2}\left(x^2 - \frac{8}{5}x\right) + 3 = \frac{5}{2}\left\{x^2 - 2 \cdot \frac{4}{5}x + \left(\frac{4}{5}\right)^2 - \frac{4^2}{5^2}\right\} - \frac{16}{3} + 3 = \frac{5}{2}\left(x - \frac{4}{5}\right)^2 - \frac{8}{5} + \frac{15}{5} \\ &= \frac{5}{2}\left(x - \frac{4}{5}\right)^2 + \frac{7}{5} .\end{aligned}$$

$$\begin{aligned}(2) \quad -\frac{8}{3}x^2 + 2x + 1 &= -\frac{8}{3}\left(x^2 - \frac{3}{4}x\right) + 1 = -\frac{8}{3}\left\{x^2 - 2 \cdot \frac{3}{8}x + \left(\frac{3}{8}\right)^2 - \frac{3^2}{8^2}\right\} + 1 = -\frac{8}{3}\left(x - \frac{3}{8}\right)^2 + \frac{3}{8} + \frac{8}{8} \\ &= -\frac{8}{3}\left(x - \frac{3}{8}\right)^2 + \frac{11}{8} .\end{aligned}$$