

## § 6.5 加法定理

**問題 6.5.1**  $\cos 60^\circ = \frac{1}{2}$ ,  $\sin 60^\circ = \frac{\sqrt{3}}{2}$ . 正弦の加法定理より,

$$\sin(\theta + 60^\circ) = \sin\theta \cos 60^\circ + \cos\theta \sin 60^\circ = \frac{3}{4} \frac{1}{2} + \frac{\sqrt{7}}{4} \frac{\sqrt{3}}{2} = \frac{3 + \sqrt{21}}{8}.$$

余弦の加法定理より,

$$\cos(\theta + 60^\circ) = \cos\theta \cos 60^\circ - \sin\theta \sin 60^\circ = \frac{\sqrt{7}}{4} \frac{1}{2} - \frac{3}{4} \frac{\sqrt{3}}{2} = \frac{\sqrt{7} - 3\sqrt{3}}{8}.$$

**問題 6.5.2**  $(\sin\theta)^2 + (\cos\theta)^2 = 1$  なので,

$$(\sin\theta)^2 = 1 - (\cos\theta)^2 = 1 - \left(\frac{3}{4}\right)^2 = 1 - \frac{9}{16} = \frac{7}{16},$$

$\sin\theta < 0$  なので

$$\sin\theta = -\sqrt{\frac{7}{16}} = -\frac{\sqrt{7}}{4}.$$

正弦の加法定理より,

$$\sin(\theta - 30^\circ) = \sin\theta \cos 30^\circ - \cos\theta \sin 30^\circ = -\frac{\sqrt{7}}{4} \frac{\sqrt{3}}{2} - \left(-\frac{3}{4}\right) \frac{1}{2} = \frac{3 - \sqrt{21}}{8}.$$

余弦の加法定理より,

$$\cos(\theta - 30^\circ) = \cos\theta \cos 30^\circ + \sin\theta \sin 30^\circ = -\frac{3}{4} \frac{\sqrt{3}}{2} + \left(-\frac{\sqrt{7}}{4}\right) \frac{1}{2} = -\frac{3\sqrt{3} + \sqrt{7}}{8}.$$

**問題 6.5.3**

$$\sin 150^\circ = \sin(60^\circ + 90^\circ) = \cos 60^\circ = \frac{1}{2}.$$

$$\cos 150^\circ = \cos(60^\circ + 90^\circ) = -\sin 60^\circ = -\frac{\sqrt{3}}{2},$$

$$\tan 150^\circ = \frac{\sin 150^\circ}{\cos 150^\circ} = \frac{1/2}{-\sqrt{3}/2} = -\frac{1}{\sqrt{3}}.$$

**問題 6.5.4**

$$\begin{aligned} \tan(\theta + 30^\circ) &= \frac{\tan\theta + \tan 30^\circ}{1 - \tan\theta \tan 30^\circ} = \frac{2 + \frac{1}{\sqrt{3}}}{1 - 2 \cdot \frac{1}{\sqrt{3}}} = \frac{2\sqrt{3} + 1}{\sqrt{3} - 2} \\ &= \frac{(2\sqrt{3} + 1)(\sqrt{3} + 2)}{(\sqrt{3} - 2)(\sqrt{3} + 2)} = \frac{2\sqrt{3}^2 + 4\sqrt{3} + \sqrt{3} + 2}{\sqrt{3}^2 - 2^2} = \frac{6 + 5\sqrt{3} + 2}{3 - 4} \\ &= -8 - 5\sqrt{3}. \end{aligned}$$

$$\begin{aligned} \tan(\theta - 30^\circ) &= \frac{\tan\theta - \tan 30^\circ}{1 + \tan\theta \tan 30^\circ} = \frac{2 - \frac{1}{\sqrt{3}}}{1 + 2 \cdot \frac{1}{\sqrt{3}}} = \frac{2\sqrt{3} - 1}{\sqrt{3} + 2} \\ &= \frac{(2\sqrt{3} - 1)(\sqrt{3} - 2)}{(\sqrt{3} + 2)(\sqrt{3} - 2)} = \frac{2\sqrt{3}^2 - 4\sqrt{3} - \sqrt{3} + 2}{\sqrt{3}^2 - 2^2} = \frac{6 - 5\sqrt{3} + 2}{3 - 4} \\ &= 5\sqrt{3} - 8. \end{aligned}$$