§6.7　三角形の面積と正弦定理

$\boxed{\text{問題 6.7.1}}$　$\sin 120^\circ = \cos 30^\circ = \dfrac{\sqrt{3}}{2}$.　三角形 ABC の面積は

$$\frac{1}{2}\cdot 3\cdot 5\cdot \sin 120^\circ = \frac{15}{2}\cdot\frac{\sqrt{3}}{2} = \frac{15}{4}\sqrt{3}\ .$$

$\boxed{\text{問題 6.7.2}}$　正弦定理より　$\dfrac{\overline{\mathrm{BC}}}{\sin\angle\mathrm{A}} = \dfrac{\overline{\mathrm{AB}}}{\sin\angle\mathrm{C}}$　なので,

$$\overline{\mathrm{BC}} = \overline{\mathrm{AB}}\frac{\sin\angle\mathrm{A}}{\sin\angle\mathrm{C}} = \sqrt{6}\,\frac{\sin 45^\circ}{\sin 60^\circ} = \sqrt{6}\,\frac{\dfrac{1}{\sqrt{2}}}{\dfrac{\sqrt{3}}{2}} = \sqrt{6}\,\frac{2}{\sqrt{6}} = 2\ .$$

$\boxed{\text{問題 6.7.3}}$

$$\angle\mathrm{B} = 180^\circ - \angle\mathrm{A} - \angle\mathrm{C} = 180^\circ - 15^\circ - 45^\circ = 120^\circ\ .$$

$$\sin 120^\circ = \sin(30^\circ + 90^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}\ .$$

正弦定理より,

$$\overline{\mathrm{AB}} = \frac{\overline{\mathrm{AC}}}{\sin\angle\mathrm{B}}\sin\angle\mathrm{C} = \frac{6}{\sin 120^\circ}\sin 45^\circ = \frac{6}{\dfrac{\sqrt{3}}{2}}\,\frac{1}{\sqrt{2}} = \frac{1}{2}\sqrt{6} = 2\sqrt{6}\ .$$

$\boxed{\text{問題 6.7.4}}$　正弦定理より　$\dfrac{\overline{\mathrm{BC}}}{\sin\angle\mathrm{A}} = \dfrac{\overline{\mathrm{AC}}}{\sin\angle\mathrm{B}}$　なので,

$$\sin\angle\mathrm{B} = \overline{\mathrm{AC}}\frac{\sin\angle\mathrm{A}}{\overline{\mathrm{BC}}} = \sqrt{3}\cdot\frac{\sin 45^\circ}{\sqrt{2}} = \frac{\sqrt{3}}{2}\ .$$

従って　$\angle\mathrm{B} = 60^\circ$　または　$\angle\mathrm{B} = 120^\circ$.

$\boxed{\text{問題 6.7.5}}$　正弦定理より　$\dfrac{\overline{\mathrm{AB}}}{\sin\angle\mathrm{C}} = \dfrac{\overline{\mathrm{BC}}}{\sin\angle\mathrm{A}}$　なので,

$$\sin\angle\mathrm{A} = \overline{\mathrm{BC}}\frac{\sin\angle\mathrm{C}}{\overline{\mathrm{AB}}} = 1\cdot\frac{\sin 135^\circ}{\sqrt{2}} = \frac{1}{2}\ .$$

従って　$\angle\mathrm{A} = 30^\circ$　または　$\angle\mathrm{A} = 150^\circ$.　$\angle\mathrm{A} \leq 180^\circ - \angle\mathrm{C} = 45^\circ$　なので,　$\angle\mathrm{A} = 30^\circ$.