§9.3　対数の性質

問題 9.3.1

$$\begin{aligned}\log_3 36+\log_3\frac{27}{4} &= \log_3(2^2 3^2)+\log_3 27-\log_3 4\\ &= \log_3 2^2+\log_3 3^2+\log_3 3^3-\log_3 2^2 = 2\log_3 2+2+3-2\log_3 2\\ &= 5\ .\end{aligned}$$

問題 9.3.2

$$\begin{aligned}\log_6\frac{27}{7}-2\log_6 9 &= \log_6 27-\log_6 7-2\log_6 3^2 = \log_6 3^3-\log_6 7-4\log_6 3\\ &= 3\log_6 3-\log_6 7-4\log_6 3 = -\log_6 7-\log_6 3\\ &= -\log_6 21\ .\end{aligned}$$

問題 9.3.3

$$\begin{aligned}\log_5\sqrt{7}-\frac{\log_5 28}{2} &= \frac{\log_5 7}{2}-\frac{\log_5(2^2\cdot 7)}{2} = \frac{\log_5 7}{2}-\frac{\log_5 2^2+\log_5 7}{2} = -\frac{2\log_5 2}{2}\\ &= -\log_5 2\ .\end{aligned}$$

問題 9.3.4

(1)
$$\log_{16}64 = \frac{\log_2 64}{\log_2 16} = \frac{\log_2 4^3}{\log_2 4^2} = \frac{3}{2}\ .$$

(2)
$$\log_{\frac{1}{9}}27 = \frac{\log_2 27}{\log_2\frac{1}{9}} = \frac{\log_2 3^3}{\log_2\frac{1}{3^2}} = \frac{3\log_2 3}{\log_2 3^{-2}} = \frac{3\log_2 3}{-2\log_2 3} = -\frac{3}{2}\ .$$

問題 9.3.5

$$\begin{aligned}\frac{\log_2 21}{3}-\log_8 49 &= \frac{\log_2(3\cdot 7)}{3}-\frac{\log_2 7^2}{3} = \frac{\log_2 3+\log_2 7-2\log_2 7}{3} = \frac{\log_2 3-\log_2 7}{3}\\ &= \frac{1}{3}\log_2\frac{3}{7}\ .\end{aligned}$$

問題 9.3.6

$$(\log_2 3)(\log_9 10) = (\log_2 3)\frac{\log_2 10}{\log_2 9} = (\log_2 3)\frac{\log_2 10}{\log_2 3^2} = (\log_2 3)\frac{\log_2 10}{2\log_2 3} = \frac{\log_2 10}{2}\ .$$